AN OPTICAL READOUT CONFIGURATION FOR ADVANCED ACOUSTIC GRAVITATIONAL WAVE DETECTORS

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Advanced configurations of acoustic gravitational wave detectors, using nested masses to obtain a wide useful bandwidth, have a very large potential sensitivity. The main problem that must be tackled in their design and construction concerns the readout of the weak vibration signal. Optical techniques are very promising, but the schemes considered up to now are limited by the thermal and radiation pressure fluctuations induced by the small interrogation area. In this work we propose and analyze a different optical configuration which allows to overcome this problem and exploit the full sensitivity, even for the recently proposed new generation of gravitational wave detectors.

1 Introduction

The research devoted to the realization of detectors for gravitational waves (gw) has seen substantial progress since the first experiments in the early 60ies. However, it is now commonly accepted that, to open the possibility of a gw astronomy, a further substantial advance in sensitivity is necessary. For what concerns acoustic detectors, some wideband configurations have been recently proposed, based on 'dual' detectors [1]. They can be implemented with two nested masses, either spheres [2] or cylinders [3], and the signal is read in the gap as the differential deformation of the two masses.

A challenging task for acoustic detectors is the readout of the weak mechanical vibrations embedding the signal. At this purpose, recent improvements in electric sensors using SQUID amplifiers are very promising [4]. Moreover, the use of optical techniques for the displacement detection was considered in the past [5] and a complete optical readout system has recently been operated on a room-temperature Weber bar [6]. Interferometric techniques well compare with the standard capacitive and inductive sensors which are used in cryogenic bars and they are excellent candidates for the necessary further extension of the sensitivity.

Optimizing the displacement sensitivity of a Fabry-Perot interferometer (FP) requires an increase of the input laser power and/or of the cavity Finesse, up to the quantum limit given by the balance between the shot-noise effects in the detection and the back-action exerted by radiation pressure. Moreover, a really quantum-limited detection can only be obtained if thermal noise is negligible.

The optical readout considered in Ref.[2] for the dual-sphere detector is based on a FP cavity with a Finesse of 10^6 and laser power of about 7 W, in order to reach the quantum limit at the design frequency of 1.3 kHz. The shot-noise limited displacement sensitivity corresponds to 10^{-45} m^2/Hz (we consider single-side spectra) and thermal noise is negligible for $Q/T > 2 \cdot 10^8$ K^{-1}, where Q is the mechanical quality factor and T the temperature of the massive resonator. This calculation only considers the global detector response for evaluating both thermal noise and radiation pressure effects, using a normal mode expansion. On the other hand, one should also take into account local effects, due to surface deformation, for both thermal noise[7,8,9,10,11] and radiation pressure[12]. Such effects strongly limit the sensitivity if the readout exploits a small part of the detector surface.

In this contribution we present and discuss an optical configuration which allows to strongly increase the interrogation surface, thus reducing the effect of local fluctuations in the readout.

2 Calculation of local fluctuations

Both thermal noise and radiation pressure effects can be calculated from the susceptibility $\chi(\omega)$ describing the mechanical response of the mirror to an exerted pressure[7,10]. From the fluctuation-dissipation theorem we obtain the spectral power of the Brownian noise

$$S_{\mathrm{Br}}(\omega) = \frac{4\,k_{\mathrm{B}}T}{\omega}\,\mathrm{Im}\,[\chi(\omega)]\ , \tag{1}$$

where k_B is the Boltzmann constant. The displacement noise due to radiation pressure fluctuations is given by

$$S_{\mathrm{rp}}(\omega) = |\chi(\omega)|^2 \left(\frac{2}{c}\right)^2 S_{\mathrm{cav}}\ , \tag{2}$$

where c is the speed of the light and S_{cav} is the noise spectral power of the radiation impinging on the mirror. If the mirror is part of a FP cavity of Finesse F and a shot-noise limited laser with power P_{in} and optical frequency ν is resonant with the cavity, we have

$$S_{\mathrm{cav}} = 2\,h\nu \left(\frac{F}{\pi}\right)^2 P_{\mathrm{in}}\,. \tag{3}$$

If we consider a single Gaussian spot on a half-infinite mirror, for the low-frequency susceptibility we can use[13]

$$|\chi^{\mathrm{single}}| = \frac{1}{\pi^{1/2}\,w}\,\frac{1-\sigma^2}{Y} \tag{4}$$

$$\mathrm{Im}\,[\chi(\omega)] \simeq \phi\,|\chi(\omega)|\,, \tag{5}$$

where σ is the Poisson coefficient, Y is the Young modulus of the mirror material, ϕ is the loss angle (we suppose $\phi \ll 1$) and w is the beam waist at the reflecting surface. From Eqs. (1)–(5) we obtain[7,8]

$$S_{\mathrm{Br}}^{\mathrm{single}}(\omega) = \frac{4\,k_{\mathrm{B}}T}{\pi^{1/2}}\,\frac{\phi}{\omega}\,\frac{1}{w}\,\frac{1-\sigma^2}{Y} \tag{6}$$

$$S_{\mathrm{rp}}^{\mathrm{single}} = \left(\frac{2(1-\sigma^2)F}{\pi^{3/2}c\,Y\,w}\right)^2 2\,h\nu\,P_{\mathrm{in}}\,. \tag{7}$$

For a FP cavity, the Brownian fluctuations on the two mirrors are independent while the fluctuations due to radiation pressure must be summed coherently obtaining (in the approximation of equal beam size on the mirrors) $S_{\mathrm{Br}}^{\mathrm{FP}} = 2S_{\mathrm{Br}}^{\mathrm{single}}$ and $S_{\mathrm{rp}}^{\mathrm{FP}} = 4S_{\mathrm{rp}}^{\mathrm{single}}$.

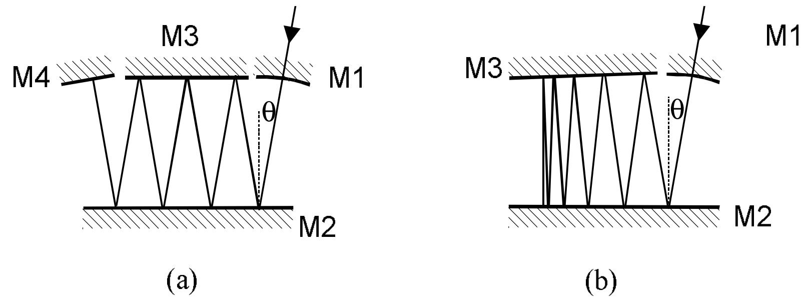

Figure 1: (a) Folded Fabry–Perot cavity with input (M1) and end (M4) mirrors, and two parallel intermediate mirrors (M2 and M3): the beam entering the cavity reflects off the intermediate mirrors with a constant incidence angle and bounces back at the end mirrors. (b) Folded Fabry–Perot cavity with an input mirror (M1) and two angled mirrors (M2 and M3): if the incidence angle of the first reflection on mirror M2 is an integer multiple n of the angle between the mirrors M2 and M3, then the beam bounces back on the same path after $n+1$ reflections.

For sapphire at 1 K we use the following material parameters: $\phi = 3 \cdot 10^{-9}$, $\sigma = 0.25$ and $Y = 4 \cdot 10^{11}$ Pa. With an input laser power of 7 W, a beam waist of 1 mm and a Finesse of 10^6, we get $S_{\rm Br}^{\rm single} = 2.7 \cdot 10^{-44}$ m^2/Hz (at 1.3 kHz) and $S_{\rm rp}^{\rm single} = 2.0 \cdot 10^{-41}$ m^2/Hz. With such parameters, both effects are larger than the sensitivity limit imposed by the shot-noise. In particular, in order to reduce Brownian noise and radiation pressure fluctuations below this limit we need a beam waist larger than, respectively, 52 mm and 280 mm.

The optical readout that we have experimented on a room temperature bar is based on a plane-spherical FP cavity with a length of 6 mm and a concave mirror radius of 6 m, giving a spot-size of about 0.18 mm. The plane-spherical geometry does not allow for great improvements, since the cavity length cannot be much increased and the dependence of the spot size on the mirror radius R is just as $R^{0.25}$. Also other cavity configurations can hardly reach a waist of few millimeters. An optical geometry alternative to the FP is given by the Herriott delay line [14], where each bounce of the beam corresponds to a different spot. Nakagawa *et al.* [15] have shown that its thermal noise effect is lower than the one of a FP with the same sensitivity. However, such an optical scheme is only suitable for low corresponding Finesse (few hundreds), while the planned Finesse of the readout is 10^6.

We have presented in a recent paper [16] a different optical configuration which permits to reach the desired low level of thermal noise and radiation pressure effects. The basic idea is to take a long FP cavity and *fold* the optical path, so that the beam experiences several reflections before getting back to the partially reflecting input mirror. This folded Fabry–Perot (FFP) maintains the sensitivity of the high Finesse FP (limited by the losses on one single mirror), but the surface fluctuations are probed by several reflections. It is possible to obtain an 'equivalent' beam radius of several centimeter while maintaining a compact cavity.

3 Design and performance of a folded Fabry–Perot

An example of FFP is shown in Fig. (1a), where the input mirror is partially reflecting and the others are high reflectors. The input and/or the end mirrors M1 and M4 are concave and the beam experiences several reflections between two intermediate flat mirrors M2 and M3 before reaching M4 and being reflected back. If we call D the distance between the mirrors M2 and M3, θ the incidence angle and N the number of bounces on M2 (there are $N-1$ bounces on M3) we get an effective cavity length $L = 2ND/\cos\theta$. The geometrical configuration of the beam, i.e., waist dimension, necessary alignment accuracy, etc., is the same as the one of a standard cavity of length L. If the input mirror transmission is T and the losses on each mirror are Σ, we have a total losses coefficient on the round-trip $\Sigma_{\rm TOT} = 4N\Sigma$. If the Pound-Drever-Hall technique[17] is used to detect the mirror displacement, the optimum signal-to-noise ratio is obtained when $T = \Sigma_{\rm TOT}$. The cavity Finesse is then $F_{\rm FFP} = 2\pi/(T + \Sigma_{\rm TOT}) = 2\pi/8N\Sigma$, i.e., a factor of $2N$ lower than the Finesse of a linear cavity. The cavity linewidth $\delta_{\rm FFP} = c/2LF_{\rm FFP}$ is just a factor of $\cos\theta$ smaller than the one of a Fabry–Perot cavity of length D made with the same mirrors (M1 and M4).

Simple geometrical considerations show that the shift $\Delta\nu$ of the resonant optical frequency ν due to a change ΔD in the position of M2 is given by $\Delta\nu/\nu = \Delta L/L = \cos^2\theta\, \Delta D/D$, i.e., for small incidence angles, it is nearly the same as in the case of a simple cavity of length D. A lower limit to the detectable signal is given by the shot noise in the detection and it is proportional to $D\delta_{\rm FFP}/\nu$. As a consequence, the sensitivity of the FFP is worse by just a factor of $\cos\theta$ with respect to the simple cavity.

We consider first the effect of the Brownian noise. If the thermal fluctuations in the mirror surface position at each spot are not correlated, then the total fluctuations sensed by the beam in a round-trip are given by $S^{2L}_{\rm Br} = (8N-2)S^{\rm single}_{\rm Br}$, corresponding to relative frequency fluctuations

$$\frac{S_\nu}{\nu^2} = \frac{S^{2L}_{\rm Br}}{(2L)^2} = \frac{S^{\rm single}_{\rm Br}}{2\,D^2}\,\frac{\cos^2\theta}{N}\left(1 - \frac{1}{4N}\right). \tag{8}$$

We have considered that the fluctuations on each spot of M2 and M3 are experienced twice in a round-trip and the two contributions must be summed coherently giving a displacement noise $4(2N-1)S^{\rm single}_{\rm Br}$, while the end mirrors are only sensed ones giving additional contribution $2S^{\rm single}_{\rm Br}$.

For a simple cavity of length D the relative frequency noise is $S_\nu/\nu^2 = S^{\rm single}_{\rm Br}/2D^2$. This is the first important result: for small incidence angles, the signal remains the same while the excess thermal noise is reduced by about a factor of N (in the power spectrum) with respect to the simple cavity.

We analyze now the radiation pressure effect. We must take into account that the radiation pressure scales with the cosine of the incidence angle and that fluctuations on all the spots sums coherently. On the other hand, the intracavity power scales with the Finesse and it is thus reduced by a factor of $2N$ with respect to the simple cavity. The fluctuations sensed in a round trip are

$$\begin{aligned} S^{2L}_{\rm rp} &= [(4N-2)\cos\theta + 2]^2 \left(\frac{1}{2N}\right)^2 S^{\rm single}_{\rm rp} \\ &= 4S^{\rm single}_{\rm rp}\cos^2\theta \left(1 + \frac{1-\cos\theta}{2N\cos\theta}\right)^2 \simeq 4S^{\rm single}_{\rm rp}\,. \end{aligned} \tag{9}$$

We remark that in the expression of Eq. (7) for $S^{\rm single}_{\rm rp}$ the Finesse F is the one of a simple cavity, made with the same mirrors of the FFP (M1 and M2). In this way the comparison is simple and the (9) shows that the fluctuations sensed in a round trip of the FFP are nearly the same as the corresponding ones in a simple cavity. The relative frequency noise is

$$\frac{S_\nu}{\nu^2} = \frac{S^{2L}_{\rm rp}}{(2L)^2} = \frac{S^{\rm single}_{\rm rp}\cos^4\theta}{4D^2N^2}\left(1 + \frac{1-\cos\theta}{2N\cos\theta}\right)^2 \simeq \frac{S^{\rm single}_{\rm rp}}{4D^2N^2}\,, \tag{10}$$

which is about a factor of $4N^2$ smaller with respect to the relative fluctuations $S_\nu/\nu^2 = S^{\rm single}_{\rm rp}/D^2$ of a simple cavity.

This is the second important result: while the Brownian noise effect decreases linearly with N, the radiation pressure effect scales even faster, as $1/N^2$.

In the above expressions one must also consider that the optical path of the FFP is much longer than the one of a simple FP. As a consequence, the beam size is larger and the values of $S^{\rm single}_{\rm Br}$ and $S^{\rm single}_{\rm rp}$ are already lower than the ones of a simple FP built with the same mirrors.

4 Calculation including spatial correlation

The approximated calculations given in the previous Section are based on the assumption that the contributions of the different spots are uncorrelated. On the other hand, Nakagawa *et al.*[15] give the expressions for the thermal noise of both a standard FP and a delay line, including space correlations, for a half-infinite space. We will now apply their formalism to the FFP, in order to check the validity of the approximated calculation developed above.

We are interested in the noise spectrum for frequencies of few kHz, i.e., well below the cutoff frequency in the response function of the Fabry–Perot which corresponds to the cavity linewidth, i.e., for $\omega < \delta_{\mathrm{FFP}}$. We remind that the linewidth of the FFP is roughly the same as the one of the simple cavity, i.e., tens of kHz. For the sake of simplicity, we will thus omit in the following the response function. We will also use the approximation of small incidence angles ($\cos\theta \simeq 1$).

The effective susceptibility of a mirror with N spots is

$$\chi_N = \chi^{\mathrm{single}} \left\{ N + 2\sum_{n=2}^{N}\sum_{q=1}^{n-1} \exp\left(-\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2}\right) I_0\left(\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2}\right) \right\}, \tag{11}$$

where $\mathbf{r}_n$ is the position of the n-th spot, w is the beam radius, assumed as constant, I_0 is the modified Bessel function of the first kind.

The Brownian noise spectrum for a FFP which has N spots on M2 and N' on M3 can be written as

$$S_{\mathrm{Br}}^{\mathrm{FFP}}(\omega) = \frac{4\,k_{\mathrm{B}}T}{\omega}\,\phi\,(4\chi_N + 4\chi_{N'} + 2\chi^{\mathrm{single}}) \tag{12}$$

and inserting the (11) we obtain

$$\begin{aligned} S_{\mathrm{Br}}^{\mathrm{FFP}}(\omega) = S_{\mathrm{Br}}^{\mathrm{single}}(\omega) \Bigg\{ & 4N + 4N' + 2 + 8\sum_{n=2}^{N}\sum_{q=1}^{n-1} \exp\left(-\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2}\right) I_0\left(\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2}\right) + \\ & 8\sum_{n'=2}^{N'}\sum_{q'=1}^{n'-1} \exp\left(-\frac{|\mathbf{r}_{n'} - \mathbf{r}_{q'}|^2}{2w^2}\right) I_0\left(\frac{|\mathbf{r}_{n'} - \mathbf{r}_{q'}|^2}{2w^2}\right) \Bigg\}. \end{aligned} \tag{13}$$

For the radiation pressure effect, we can write

$$S_{\mathrm{rp}}^{\mathrm{FFP}} = \left(\frac{2}{c}\right)^2 S_{\mathrm{cav}} \left|2\chi_N + 2\chi_{N'} + 2\chi^{\mathrm{single}}\right|^2 \tag{14}$$

and by inserting the (11) and the (3) and using the (7) (where the Finesse is the one of the corresponding simple cavity) we obtain

$$\begin{aligned} S_{\mathrm{rp}}^{\mathrm{FFP}} = S_{\mathrm{rp}}^{\mathrm{single}} \left(\frac{1}{(2N)^2}\right) \Bigg| & 4N + 4N' + 2 + 4\sum_{n=2}^{N}\sum_{q=1}^{n-1} \exp\left(-\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2}\right) I_0\left(\frac{|\mathbf{r}_n - \mathbf{r}_q|^2}{2w^2}\right) \\ & +4\sum_{n'=2}^{N'}\sum_{q'=1}^{n'-1} \exp\left(-\frac{|\mathbf{r}_{n'} - \mathbf{r}_{q'}|^2}{2w^2}\right) I_0\left(\frac{|\mathbf{r}_{n'} - \mathbf{r}_{q'}|^2}{2w^2}\right) \Bigg|^2. \end{aligned} \tag{15}$$

The effect of the interaction between surface fluctuations at different spots can be analyzed directly from the expressions of χ. The right-hand-side of Eq. (11) includes a first term,

$$\chi_{\mathrm{u}} = N\chi^{\mathrm{single}},$$

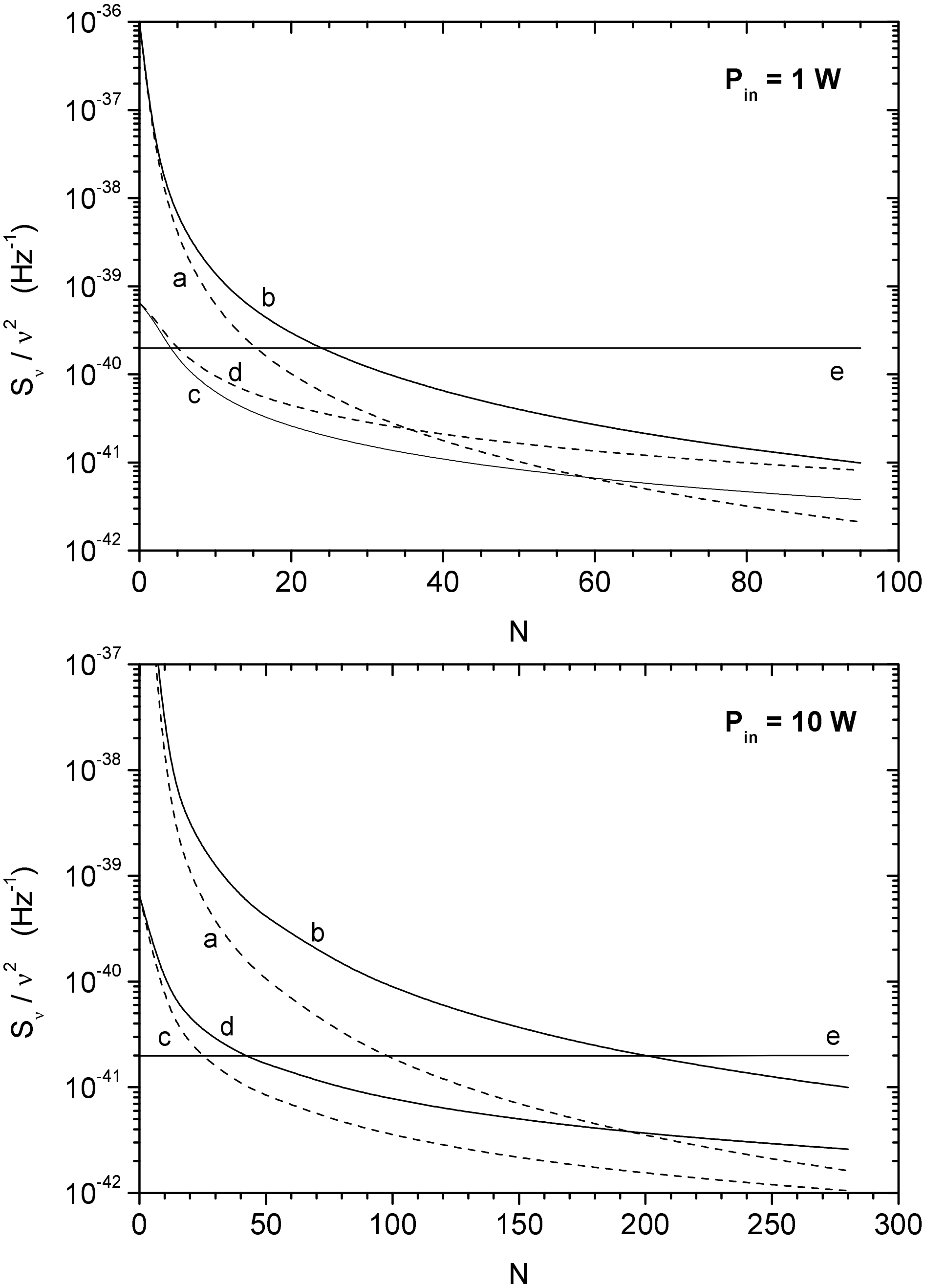

Figure 2: Relative frequency power noise level S_ν/ν^2 as a function of the number of spots N on a FFP, for a distance $d = 4w$ between spots and for two different value of impinging power $P_{\rm in} = 1$ W and 10 W. (a) radiation pressure noise without correlation terms; (b) radiation pressure noise with correlation terms; (c) Brownian noise (at 1300 Hz) without correlation terms; (d) Brownian noise (at 1300 Hz) with correlation terms; (e) shot noise effect in the Pound-Drever detection. The values at $N = 0$ correspond to the case of a simple FP cavity. The calculation is performed with: $F = 10^6$, $R = 10$ m, $D = 6$ mm and the parameters of Sapphire at 1 K.

corresponding to uncorrelated noise sources in the different spots, while the remaining terms with double summations accounts for the extra noise due to correlation between spots on the same mirror. The first term gives the noise spectra that we have found in the previous approximated calculations.

The additional noise due to correlation rapidly decreases when d gets higher than the beam radius. Geometrical constraints require a distance between spots of a least $d = 4w_0$. In this case, the susceptibility results to be correct within a factor of three[16] and the simple expressions derived are an useful tool for a quick estimate of the possible performance and the conceptual validity of the FFP is confirmed by the calculation which includes the correlation terms.

The final result is better express by the relative frequency fluctuations, that can be directly compared with the shot noise. In Fig. (2) we report S_ν/ν^2 for a FFP with N spots on M2 and $N' = N - 1$ spots on M3, as a function of N, for $P_{\rm in} = 1$ W and 10 W. The approximated expressions (8) and (10) (with $\cos\theta = 1$) are compared with the results which take into account the correlations between spots, using the complete expressions for the noise densities (13) and (15). The limits corresponding to a simple FP are given by the extrapolation at $N = 0$ of the respective curves. The shot-noise levels are also reported. The improvement allowed by the FFP is clearly appreciated, as well as the possibility to reach a quantum-limited detection.

5 Conclusion

We have presented an optical cavity configuration which allows to strongly reduce thermal and back-action fluctuations related to the small sensing area of standard cavities.

In this study we have used the half-infinite space approximation for calculating the susceptibility. This assumption is valid as soon as the dimension of the surface interrogated is small with respect to the one of the overall detector and, within this limit, our calculation has a general validity and it does not depend on the detector shape.

A more accurate calculation of the detector performance for large interrogation areas cannot distinguish between 'global' and 'local' effects and one must consider the exact susceptibility of each particular detector and read-out configuration. Calculation methods to accurately evaluate it are presently being developed[18]. However, the behavior of the noise reduction predicted in this work allows to closely approach with the present technology the quantum-limited sensitivity calculated for the main mode of the detector, overcoming the problem of the excess noise related to the small beam radius. Such a result is crucial for the design of the next generation of gravitational wave detectors.

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